Correlations of eigenfunctions in disordered systems

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Correlations of eigenfunctions, $\langle|\psi_k(\boldsymbol{r}_1)|^2|\psi_l(\boldsymbol{r}_2)|^2\rangle$, in a disordered system are investigated. We derive general formulas expressing these correlation functions in terms of the supermatrix σ model. In the particular case of the weak localization regime we find that the correlations of the same eigenfunction are proportional to g^{-1} for large distances, while the correlations of two different eigenfunctions cross over from g^{-1} behavior for $\boldsymbol{r}_1=\boldsymbol{r}_2$ to g^{-2} behavior for $|\boldsymbol{r}_1-\boldsymbol{r}_2|\gg l$, with g and l being the dimensionless conductance and the mean free path, respectively. [S1063-651X(97)12805-7]

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Statistics of eigenfunction fluctuations in disordered and chaotic systems have attracted research interest recently. The fluctuations of eigenfunction amplitudes determine statistical properties of conductance peaks and level widths in quantum dots in the Coulomb blockade regime [1–5], and can be directly measured in the microwave cavity experiments [6,7]. On the theoretical side, the recent progress is based on application of the supersymmetry method to the problem of eigenfunction statistics [8,2]. It was found that the distribution of eigenfunction amplitudes is in the leading (zero-mode) approximation correctly described by formulas of the random matrix theory (RMT). Deviations from the RMT predictions were studied in Refs. [8–10]. Correlations of amplitudes of an eigenfunction in two different spatial points were considered in Ref. [11] on the level of zero-mode approximation; the latter was shown [12] to be equivalent to the RMT-like assumption of the Gaussian fluctuations of wave functions.

All results mentioned above concern fluctuations of the *same* eigenfunction. In the present paper we study correlations of amplitudes of two *different* eigenfunctions. We derive general expressions in terms of the supermatrix σ model, valid for arbitrary diffusive (or classically chaotic) system, and then apply them to the weak localization regime.

In order to evaluate the correlations of the wave functions we use a technique similar to that of Ref. [13]. Namely, we consider a quantity

$$A(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)=\left\langle \sum_{k,l} |\psi_k(\boldsymbol{r}_1)\psi_l(\boldsymbol{r}_2)|^2 \times \delta(\epsilon-\epsilon_k)\,\delta(\epsilon+\omega-\epsilon_l)\right\rangle - \left\langle \sum_k |\psi_k(\boldsymbol{r}_1)|^2\delta(\epsilon-\epsilon_k)\right\rangle \times \left\langle \sum_l |\psi_l(\boldsymbol{r}_2)|^2\delta(\epsilon+\omega-\epsilon_l)\right\rangle. \quad (1)$$

Here the angular brackets denote the impurity average. We have introduced the eigenstates $\psi_k(\boldsymbol{r})$ and eigenvalues ϵ_k of the Hamiltonian $\hat{H}_0=\hat{H}_0+U(\boldsymbol{r})$ in a particular disorder configuration $U(\mathbf{r})$, $\hat{H}_0$ being the Hamiltonian of the free particle. Further we define the eigenfunction correlators

$$\alpha(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon)=\langle|\psi_k(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)|^2\rangle_\epsilon \equiv \frac{\left\langle \sum_k \left|\psi_k(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)\right|^2\delta(\epsilon-\epsilon_k)\right\rangle}{\left\langle \sum_k \delta(\epsilon-\epsilon_k)\right\rangle},$$

$$\beta(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega) =\langle|\psi_k(\boldsymbol{r}_1)\psi_l(\boldsymbol{r}_2)|^2\rangle_{\epsilon,\omega}, \quad k\neq l \equiv \frac{\left\langle \sum_{k\neq l} \left|\psi_k(\boldsymbol{r}_1)\psi_l(\boldsymbol{r}_2)\right|^2\delta(\epsilon-\epsilon_k)\,\delta(\epsilon+\omega-\epsilon_l)\right\rangle}{\left\langle \sum_{k\neq l} \delta(\epsilon-\epsilon_k)\,\delta(\epsilon+\omega-\epsilon_l)\right\rangle}. \quad (2)$$

One can then rewrite Eq. (1) as

$$A(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)=\alpha(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon)\Delta^{-1}\delta(\omega) +\beta(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)\Delta^{-2}R(\omega)-\nu^2, \quad (3)$$

where Δ is the mean level spacing, $\Delta=(\nu V)^{-1}$, with V and ν being the system volume and the density of states, respectively. Here we have introduced the two-level correlation function

$$R(\omega)=\Delta^2\left\langle \sum_{k\neq l} \delta(\epsilon-\epsilon_k)\,\delta(\epsilon+\omega-\epsilon_l)\right\rangle. \quad (4)$$

We would like to stress that we *do not* assume any decoupling of eigenfunction and eigenvalue correlations.

On the other hand, the quantity (1) can be written in terms of the Green's functions in the coordinate-frequency representation

$$A(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)=(2\pi^2)^{-1}\mathrm{Re}\{\langle G^R(\boldsymbol{r}_1,\boldsymbol{r}_1,\epsilon)G^A(\boldsymbol{r}_2,\boldsymbol{r}_2,\epsilon+\omega)\rangle - \langle G^R(\boldsymbol{r}_1,\boldsymbol{r}_1,\epsilon)\rangle\langle G^A(\boldsymbol{r}_2,\boldsymbol{r}_2,\epsilon+\omega)\rangle\}. \quad (5)$$

The expression (5) can be directly calculated with the use of the supersymmetry technique [14–16]. We concentrate in the sequel on the case of broken time-reversal symmetry (unitary ensemble); generalization to the other ensembles is straightforward. After the standard manipulations we get

$$A(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)=-(2\pi^2)^{-1}\mathrm{Re}\{\langle g^{11}_{bb}(\boldsymbol{r}_1,\boldsymbol{r}_1)g^{22}_{bb}(\boldsymbol{r}_2,\boldsymbol{r}_2) + g^{12}_{bb}(\boldsymbol{r}_1,\boldsymbol{r}_2)g^{21}_{bb}(\boldsymbol{r}_2,\boldsymbol{r}_1)\rangle_F - \langle g^{11}_{bb}(\boldsymbol{r}_1,\boldsymbol{r}_1)\rangle_F\langle g^{22}_{bb}(\boldsymbol{r}_2,\boldsymbol{r}_2)\rangle_F\}. \quad (6)$$

Here $\langle\cdots\rangle_F$ denotes the averaging with the action of the supermatrix σ model $F[Q]$:

$$\langle\cdots\rangle_F=\int DQ(\cdots)\exp(-F[Q]),$$

$$F[Q]=-\frac{\pi\nu}{4}\int d\boldsymbol{r}\ \mathrm{Str}[D(\nabla Q)^2+2i(\omega+i0)\Lambda Q], \quad (7)$$

where D is the diffusion coefficient, $Q=T^{-1}\Lambda T$ is a 4×4 supermatrix, $\Lambda=\mathrm{diag}(1,1,-1,-1)$, and T belongs to the supercoset space $U(1,1|2)/U(1|1)\times U(1|1)$. The symbol Str denotes the supertrace defined as $\mathrm{Str}(B)=B^{11}_{bb}-B^{11}_{ff}+B^{22}_{bb}-B^{22}_{ff}$. The upper matrix indices correspond to the retarded-advanced decomposition, while the lower indices denote the boson-fermion one. The Green's function g in Eq. (6) is the solution to the matrix equation

$$\left[-i\left(\epsilon+\frac{\omega}{2}-\hat{H}_0\right)-\frac{i}{2}(\omega+i0)\Lambda+Q/2\tau\right]g(\boldsymbol{r},\boldsymbol{r}')=\delta(\boldsymbol{r}-\boldsymbol{r}'). \quad (8)$$

Expressing these functions through the matrices Q and taking into account Eq. (3), we arrive at the following equation valid in an arbitrary diffusive system:

$$2\pi^2\left[\frac{\alpha(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon)}{\Delta}\delta(\omega)+\frac{\beta(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)}{\Delta^2}R(\omega)\right] = -(\pi\nu)^2\mathrm{Re}\langle Q^{11}_{bb}(\boldsymbol{r}_1)Q^{22}_{bb}(\boldsymbol{r}_2)\rangle_F - [\mathrm{Im}\ G^R(\boldsymbol{r}_1-\boldsymbol{r}_2)]^2\mathrm{Re}\langle Q^{12}_{bb}(\boldsymbol{r}_1)Q^{21}_{bb}(\boldsymbol{r}_1)\rangle_F+(\pi\nu)^2, \quad (9)$$

with G^R being the impurity averaged retarded Green's function. In particular, in the case of the 2D and 3D system, G^R is given by

$$G^R(\boldsymbol{r})=\begin{cases}-i\nu\displaystyle\int_{-\pi/2}^{\pi/2}d\theta\exp[(ip_Fr-r/2l)\cos\theta], & 2\mathrm{D}\\ -\pi\nu(p_Fr)^{-1}\exp[ip_Fr-r/2l], & 3\mathrm{D},\end{cases}$$

where l is the mean free path. When deriving the second term on the right-hand side (rhs) of Eq. (9), we approximated the field $Q(\boldsymbol{r})$ in Eq. (8) by a constant matrix $Q(\boldsymbol{r}_1)$. This approximation works both for $r=|\boldsymbol{r}_1-\boldsymbol{r}_2|\ll l$ (due to slow variation of the Q field) and for $r\gg l$ [when the Green's function $g(\boldsymbol{r}_1,\boldsymbol{r}_2)$ is exponentially small anyway, and the second term on the rhs of Eq. (9) is negligibly small].

The key point making further progress possible is that the term containing the single eigenfunction correlations on the lhs of Eq. (9) is proportional to $\delta(\omega)$, whereas the one depending on the correlations of two different eigenfunctions is regular at $\omega=0$. Thus, separation of the expression on the rhs of Eq. (9) into the singular [proportional to $\delta(\omega)$] and regular parts allows one to obtain the quantities $\alpha(\boldsymbol{r}_1,\boldsymbol{r}_2)$ and $\beta(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega)$.

Now we turn to the case of a metallic system in the weak localization regime. The corresponding small parameter is given by Eq. (19) below. For further purposes, we introduce the functions

$$f_1(\boldsymbol{r}_1,\boldsymbol{r}_2)=\Pi^2(\boldsymbol{r}_1,\boldsymbol{r}_2),$$

$$f_2(\boldsymbol{r}_1,\boldsymbol{r}_2)=(2V)^{-1}\int d\boldsymbol{r}[\Pi^2(\boldsymbol{r},\boldsymbol{r}_1)+\Pi^2(\boldsymbol{r},\boldsymbol{r}_2)], \quad (10)$$

$$f_3=V^{-2}\int d\boldsymbol{r}d\boldsymbol{r}'\Pi^2(\boldsymbol{r},\boldsymbol{r}'),$$

$$f_4(\boldsymbol{r}_1,\boldsymbol{r}_2)=V^{-1}\int d\boldsymbol{r}\Pi(\boldsymbol{r},\boldsymbol{r}_1)\Pi(\boldsymbol{r},\boldsymbol{r}_2).$$

Here the diffusion propagator Π is the solution to the diffusion equation

$$-D\nabla^2\Pi(\boldsymbol{r}_1,\boldsymbol{r}_2)=(\pi\nu)^{-1}[\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)-V^{-1}] \quad (11)$$

with appropriate boundary conditions. We obtain

$$\Pi(\boldsymbol{r}_1,\boldsymbol{r}_2)=(\pi\nu)^{-1}\sum_{\boldsymbol{q}}(Dq^2)^{-1}\phi_{\boldsymbol{q}}(\boldsymbol{r}_1)\phi_{\boldsymbol{q}}(\boldsymbol{r}_2), \quad (12)$$

with $\phi_{\boldsymbol{q}}$ being the eigenfunction of the diffusion operator corresponding to the eigenvalue Dq^2, $\boldsymbol{q}\neq\boldsymbol{0}$. The level correlation function has the form [17]

$$R(\omega)=1-s^{-2}\sin^2s+f_3\sin^2s+O(g^{-3}), \quad (13)$$

where a dimensionless parameter $s=\pi\omega/\Delta$ is introduced. The first two terms in Eq. (13) are given by RMT, while the third one is the correction of order g^{-2} due to the diffusion modes. Here $g=2\pi E_c/\Delta$ is the dimensionless conductance, with E_c being the Thouless energy.

The σ model correlation functions $\langle Q^{11}_{bb}(\boldsymbol{r}_1)Q^{22}_{bb}(\boldsymbol{r}_2)\rangle_F$ and $\langle Q^{12}_{bb}(\boldsymbol{r}_1)Q^{21}_{bb}(\boldsymbol{r}_2)\rangle_F$ can be calculated for relatively low frequencies $\omega\ll E_c$ with the use of a general method developed in Refs. [17,9], which allows one to take into account spatial variations of the field Q. The results are obtained in form of expansions in g^{-1}. First, we restrict ourselves to the terms of order g^{-1}. Then, the result for the first correlator reads as

$$\langle Q^{11}_{bb}(\boldsymbol{r}_1)Q^{22}_{bb}(\boldsymbol{r}_2)\rangle_F=-1-2i\frac{\exp(is)\sin s}{(s+i0)^2} - 2i\frac{1}{s+i0}\Pi(\boldsymbol{r}_1,\boldsymbol{r}_2). \quad (14)$$

The first two terms in Eq. (14) represent the result of the so-called zero-mode approximation [14], which takes into account only the spatially constant configurations of the field $Q(\boldsymbol{r})$, so that the functional integral over $DQ(\boldsymbol{r})$ is reduced to an integral over a single matrix Q. The last term is the correction of order g^{-1}. An analogous calculation for the second correlator yields [13]

$$\langle Q_{bb}^{12}(\boldsymbol{r}_1)Q_{bb}^{21}(\boldsymbol{r}_2)\rangle_F = -2\left\{\frac{i}{s+i0}+\left[1+i\frac{\exp(is)\sin s}{(s+i0)^2}\right]\times\Pi(\boldsymbol{r}_1,\boldsymbol{r}_2)\right\}. \quad (15)$$

Now, separating regular and singular parts on the rhs of Eq. (9), we obtain the following result for the autocorrelations of the same eigenfunction:

$$V^2\langle|\psi_k(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)|^2\rangle_\epsilon - 1 = k_d(r)[1+\Pi(\boldsymbol{r}_1,\boldsymbol{r}_1)]+\Pi(\boldsymbol{r}_1,\boldsymbol{r}_2), \quad (16)$$

and for the correlation of amplitudes of two different eigenfunctions

$$V^2\langle|\psi_k(\boldsymbol{r}_1)\psi_l(\boldsymbol{r}_2)|^2\rangle_{\epsilon,\omega} - 1 = k_d(r)\Pi(\boldsymbol{r}_1,\boldsymbol{r}_1), \quad k\neq l. \quad (17)$$

Here the function $k_d(r)$ is defined as

$$k_d(r) = (\pi\nu)^{-2}[\mathrm{Im}G^R(\boldsymbol{r})]^2 = e^{-r/l}\begin{cases}1, & \text{2D}, \quad p_F r \ll 1\\ 2(\pi p_F r)^{-1}\cos^2(p_F r-\pi/4), & \text{2D}, \quad p_F r \gg 1\\ (p_F r)^{-2}\sin^2 p_F r, & \text{3D}.\end{cases}$$

In particular, for $\boldsymbol{r}_1=\boldsymbol{r}_2$ we have

$$V^2\langle|\psi_k(\boldsymbol{r})\psi_l\boldsymbol{r})|^2\rangle_{\epsilon,\omega} - 1 = \delta_{kl}+(1+\delta_{kl})\Pi(\boldsymbol{r},\boldsymbol{r}). \quad (18)$$

Note that the result (16) for $\boldsymbol{r}_1=\boldsymbol{r}_2$ is the inverse participation ratio previously obtained in Ref. [9], while that for arbitrary spatial separation was found in the zero-mode approximation ($g=\infty$) in Ref. [11].

Equations (17) and (18) show that the correlations between different eigenfunctions are relatively small in the weak disorder regime. Indeed, they are proportional to the small parameter $\Pi(\boldsymbol{r},\boldsymbol{r})$, which is equal in the case of 2D geometry to (L is the size of the system)

$$\Pi(\boldsymbol{r},\boldsymbol{r}) = (\pi g)^{-1}\ln L/l, \quad \text{2D}. \quad (19)$$

For a quasi-1D wire or strip of the length L,

$$\Pi(r,r) = \frac{2}{g}\left[\frac{1}{6}+B_2\left(\frac{r}{L}\right)\right], \quad 0\leq r\leq L, \quad (20)$$

where $B_2(x)=x^2-x+1/6$ is the Bernoulli polynomial [18]. The correlations are enhanced by disorder; when the system approaches the strong localization regime, the relative magnitude of correlations, $\Pi(\boldsymbol{r},\boldsymbol{r})$, becomes the quantity of order of unity.

An inspection of Eqs. (16) and (17) shows that while the correlations of amplitude of the same wave function survive for the large separation between the points, $r\gg l$, and are proportional to g^{-1}, the correlations of two different wave functions decay exponentially for the distances larger than the mean free path l. This is, however, an artifact of the g^{-1} approximation, and the investigation of the corresponding tails requires the extension of the above calculation to the terms proportional to g^{-2}. We find that the correlator $\langle Q_{bb}^{11}Q_{bb}^{22}\rangle_F$ gets the following correction:

$$\delta\langle Q_{bb}^{11}Q_{bb}^{22}\rangle_F = -f_1+2f_4+\exp(2is)f_3 - 2i\frac{\exp(2is)}{s+i0}(f_2-f_3) - \frac{\exp(2is)-1}{2(s+i0)^2}(f_1-4f_2+3f_3-2f_4). \quad (21)$$

Consequently, we obtain the following results for the correlations of different ($k\neq l$) eigenfunctions at $r>l$:

$$V^2\langle|\psi_k(\boldsymbol{r}_1)\psi_l(\boldsymbol{r}_2)|^2\rangle_{\epsilon,\omega} - 1 = \frac{1}{2}(f_1-f_3-2f_4)+2(f_2-f_3)\times\left(\frac{\sin^2 s}{s^2}-\frac{\sin 2s}{2s}\right)\times\left(1-\frac{\sin^2 s}{s^2}\right)^{-1}. \quad (22)$$

As it should be expected, the double integral over both coordinates of this correlation function is equal to zero. This property is just the normalization condition and should hold in arbitrary order of expansion in g^{-1}.

The quantities f_2, f_3, and f_4 are proportional to g^{-2}, with some prefactors of order unity [17]. On the other hand, f_1 in 2D and 3D geometry depends essentially on the distance $r=|\boldsymbol{r}_1-\boldsymbol{r}_2|$. In particular, for $l\ll r\ll L$ we find

$$f_1(\boldsymbol{r}_1,\boldsymbol{r}_2)=\Pi^2(\boldsymbol{r}_1,\boldsymbol{r}_2)\approx\begin{cases}\dfrac{1}{(\pi g)^2}\ln^2\dfrac{L}{r}, & \text{2D}\\ \dfrac{1}{(4\pi^2\nu D r)^2}, & \text{3D.}\end{cases}$$

Thus, for $l<r\ll L$, the contributions proportional to f_1 dominate in Eq. (22), and we get

$$V^2\langle|\psi_k(\boldsymbol{r}_1)\psi_l(\boldsymbol{r}_2)|^2\rangle_{\epsilon,\omega}-1=\frac{1}{2}\Pi^2(\boldsymbol{r}_1,\boldsymbol{r}_2),\quad k\neq l. \tag{23}$$

On the other hand, for the case of quasi-1D geometry (as well as in 2D and 3D for $r\sim L$), all quantities f_1, f_2, f_3, and f_4 are of order of $1/g^2$. Thus, the correlator (22) acquires a nontrivial frequency dependence on a scale $\omega\sim\Delta$ described by the second term on the rhs of Eq. (22). In particular, in the quasi-1D case

$$f_2-f_3=-\frac{2}{3g^2}\left[B_4\left(\frac{r_1}{L}\right)+B_4\left(\frac{r_2}{L}\right)\right], \tag{24}$$

where $B_4(x)=x^4-2x^3+x^2-1/30$. Therefore, we find a coupling of the eigenfunction and eigenvalue statistics.

Another correlation function, generally used for the calculation of the linear response of the system,

$$\gamma(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)=\langle\psi_k^*(\boldsymbol{r}_1)\psi_l(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)\psi_l^*(\boldsymbol{r}_2)\rangle_{\epsilon,\omega},\quad k\neq l,$$

can be calculated in a similar way (cf. [13]). Starting from the quantity

$$\begin{aligned}B(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)=&\left\langle\sum_{k,l}\psi_k^*(\boldsymbol{r}_1)\psi_l(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)\psi_l^*(\boldsymbol{r}_2)\right.\\&\left.\times\delta(\epsilon-\epsilon_k)\,\delta(\epsilon+\omega-\epsilon_l)\right\rangle\\&-\left\langle\sum_k\psi_k^*(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)\,\delta(\epsilon-\epsilon_k)\right\rangle\\&\times\left\langle\sum_l\psi_l(\boldsymbol{r}_1)\psi_l^*(\boldsymbol{r}_2)\,\delta(\epsilon+\omega-\epsilon_l)\right\rangle,\end{aligned} \tag{25}$$

and repeating the derivation that led us to Eq. (9), we get another identity:

$$\begin{aligned}&2\pi^2\left[\frac{\alpha(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon)}{\Delta}\,\delta(\omega)+\frac{\gamma(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)}{\Delta^2}R(\omega)\right]\\&\quad=-(\pi\nu)^2\mathrm{Re}\{\langle Q_{bb}^{12}(\boldsymbol{r}_1)Q_{bb}^{21}(\boldsymbol{r}_2)\rangle_F\\&\qquad+k_d(r)\langle Q_{bb}^{11}(\boldsymbol{r}_1)Q_{bb}^{22}(\boldsymbol{r}_1)\rangle_F-k_d(r)\},\end{aligned} \tag{26}$$

Taking into account Eqs. (14) and (15), and separating the rhs into the regular and singular parts, we recover Eq. (16) and obtain

$$V^2\langle\psi_k^*(\boldsymbol{r}_1)\psi_l(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)\psi_l^*(\boldsymbol{r}_2)\rangle_{\epsilon,\omega}=k_d(r)+\Pi(\boldsymbol{r}_1,\boldsymbol{r}_2),\quad k\neq l. \tag{27}$$

As was mentioned, the above derivation is valid for $\omega\ll E_c$. In order to obtain the results in the range $\omega\gtrsim E_c$ one can calculate the σ model correlation functions entering Eqs. (9) and (26) by means of the perturbation theory [19]. We find then for $k\neq l$

$$\begin{aligned}V^2\langle|\psi_k(\boldsymbol{r}_1)\psi_l(\boldsymbol{r}_2)|^2\rangle_{\epsilon,\omega}=1+\mathrm{Re}\Bigg\{&k_d(r)\Pi_\omega(\boldsymbol{r}_1,\boldsymbol{r}_2)\\&+\frac{1}{2}\Bigg[\Pi_\omega^2(\boldsymbol{r}_1,\boldsymbol{r}_2)\\&-\frac{1}{V^2}\int d\boldsymbol{r}d\boldsymbol{r}'\Pi_\omega^2(\boldsymbol{r},\boldsymbol{r}')\Bigg]\Bigg\},\end{aligned}$$

$$V^2\langle\psi_k^*(\boldsymbol{r}_1)\psi_l(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)\psi_l^*(\boldsymbol{r}_2)\rangle_{\epsilon,\omega}=k_d(r)+\mathrm{Re}\Pi_\omega(\boldsymbol{r}_1,\boldsymbol{r}_2), \tag{28}$$

where $\Pi_\omega(\boldsymbol{r}_1,\boldsymbol{r}_2)$ is the finite-frequency diffusion propagator

$$\Pi_\omega(\boldsymbol{r}_1,\boldsymbol{r}_2)=(\pi\nu)^{-1}\sum_{\boldsymbol{q}}\frac{\phi_q(\boldsymbol{r}_1)\phi_q(\boldsymbol{r}_2)}{Dq^2-i\omega}, \tag{29}$$

and the summation in Eq. (29) now includes $\boldsymbol{q}=\boldsymbol{0}$.

Finally, we discuss the relation between our results and experiments carried out on quantum dots in the Coulomb blockade regime. Our theory predicts only weak [$\sim\Pi(\boldsymbol{r},\boldsymbol{r})$] correlations of the amplitudes of different wave functions in the same point, which implies weak correlations of neighboring conductance peak heights. Indeed, this is in agreement with the experiments [20,4]. On the other hand, strong correlations of amplitudes were observed recently [5]. In principle, one could imagine that the dot was far from the universal (RMT) regime, so that the parameter $\Pi(\boldsymbol{r},\boldsymbol{r})$ was not small. However, this would be in contradiction with the fact that the total distribution of peak heights in Ref. [5] was well described by the RMT formulas [1], since the corrections to the distribution of $|\psi^2(\boldsymbol{r})|$ (and consequently to that of peak heights) are proportional to the same parameter $\Pi(\boldsymbol{r},\boldsymbol{r})$ [9]. Possibly, variation of the peak heights in the experiment [5] may have another source on top of the fluctuations of the eigenfunctions.

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[1] R. A. Jalabert, A. D. Stone, and Y. Alhassid, Phys. Rev. Lett. **68**, 3468 (1992).

[2] V. N. Prigodin, K. B. Efetov, and S. Iida, Phys. Rev. Lett. **71**, 1230 (1993).

[3] E. R. Mucciolo, V. N. Prigodin, and B. L. Altshuler, Phys. Rev. B **51**, 1714 (1995).

[4] A. M. Chang, H. U. Baranger, L. N. Pfeiffer, K. W. West, and T. Y. Chang, Phys. Rev. Lett. **76**, 1695 (1996).

[5] J. A. Folk, S. R. Patel, S. F. Godijn, A. G. Huibers, S. M. Cronenwett, C. M. Marcus, K. Campman, and A. C. Gossard, Phys. Rev. Lett. **76**, 1699 (1996).
[6] H. J. Stöckmann and J. Stein, Phys. Rev. Lett. **64**, 2215 (1990); J. Stein and H. J. Stöckmann, *ibid.* **68**, 2867 (1992).
[7] S. Sridhar, Phys. Rev. Lett. **67**, 785 (1991); A. Kudrolli, V. Kidambi, and S. Sridhar, *ibid.* **75**, 822 (1995).
[8] A. D. Mirlin and Y. V. Fyodorov, J. Phys. A **26**, L551 (1993).
[9] Y. V. Fyodorov and A. D. Mirlin, Pis'ma Zh. Eksp. Teor. Fiz. **60**, 779 (1994) [JETP Lett. **60**, 790 (1994)]; Phys. Rev. B **51**, 13 403 (1995).
[10] V. I. Fal'ko and K. B. Efetov, Phys. Rev. B **52**, 17 413 (1995); A. D. Mirlin, J. Math. Phys. **38**, 1888 (1997).
[11] V. N. Prigodin, Phys. Rev. Lett. **74**, 1566 (1995); V. N. Prigodin, N. Taniguchi, A. Kudrolli, V. Kidambi, and S. Sridhar, *ibid.* **75**, 2392 (1995).
[12] M. Srednicki, Phys. Rev. E **54**, 954 (1996); M. Srednicki and F. Stiernelof, J. Phys. A **29**, 5817 (1996).
[13] Ya. M. Blanter and A. D. Mirlin, Phys. Rev. B **53**, 12 601 (1996).
[14] K. B. Efetov, Adv. Phys. **32**, 53 (1983).
[15] J. J. M. Verbaarschot, H. A. Weidenmüller, and M. R. Zirnbauer, Phys. Rep. **129**, 367 (1985).
[16] M. R. Zirnbauer, Nucl. Phys. **B265**, 375 (1986).
[17] V. E. Kravtsov and A. D. Mirlin, Pis'ma Zh. Eksp. Teor. Fiz. **60**, 645 (1994) [JETP Lett. **60**, 656 (1994)].
[18] In the 3D geometry, the sum over the momenta q in Eq. (12) determining $\Pi(\boldsymbol{r},\boldsymbol{r})$ diverges at large q and is deteremined by the upper cutoff, $q\sim 1/l$, yielding $\Pi(\boldsymbol{r},\boldsymbol{r})\sim g^{-1}L/l$. This reflects the fact that in 3D geometry the truly local $(\boldsymbol{r}_1=\boldsymbol{r}_2)$ correlations may not be given correctly by the diffusion approximation and can depend on microscopic structure of the random potential. For this reason, we do not consider local correlations in 3D geometry here. Note, however, that this concerns the global geometry of the sample; locally the system can be either of 2D or 3D nature, which determines the form of the function $k_d(r)$ (e.g., a wire is locally 3D, but has a quasi-1D geometry).
[19] B. L. Altshuler and B. I. Shklovskii, Zh. Eksp. Teor. Fiz. **91**, 220 (1986) [Sov. Phys. JETP **64**, 127 (1986)].
[20] E. B. Foxman *et al.*, Phys. Rev. B **50**, 14 193 (1994).